

Ch. 6 Review

$$1. f(x) = 4^x$$

$$\begin{aligned}f'(x) &= 4^x \ln 4 \\&= (\ln 4) 4^x\end{aligned}$$

$$2. f(x) = 5^{-4x}$$

$$\begin{aligned}f'(x) &= 5^{-4x} (\ln 5) (-4) \\&= (-4 \ln 5) 5^{-4x}\end{aligned}$$

$$3. f(x) = x^{9x}$$

$$\begin{aligned}f'(x) &= x(\ln 9) 9^x + 9^x \\f'(x) &= 9^x(x \ln 9 + 1)\end{aligned}$$

$$4. f(x) = 2^{-\theta} \cos \theta$$

$$\begin{aligned}f'(x) &= 2^{-\theta} (-\pi \sin \theta) - \cos \theta 2^{-\theta} \ln 2 \\&= (-\pi \sin \theta) 2^{-\theta} - \cos \theta (\ln 2) 2^{-\theta} \\&= 2^{-\theta} (\pi \sin \theta + (\ln 2) \cos \theta)\end{aligned}$$

$$5. y = \log_4(5x+1)$$

$$y' = \frac{1}{\ln 4} \frac{1}{5x+1} \cdot 5$$

$$y' = \frac{5}{(\ln 4)(5x+1)}$$

$$6. h(t) = \log_5(4-t)^2$$

$$h(t) = 2 \log_5(4-t)$$

$$h'(t) = \frac{-2}{(\ln 5)(4-t)}$$

$$7. h(t) = \log_5 \sqrt{x^2-1}$$

$$h(t) = \frac{1}{2} \log_5(x^2-1)$$

$$h'(t) = \frac{-x}{(\ln 5)(x^2-1)}$$

$$8. h(t) = \log_2 \frac{x^2}{x-1}$$

$$h(t) = 2 \log_2 x - \log_2(x-1)$$

$$h'(t) = \frac{2}{x \ln 2} - \frac{1}{\ln 2(x-1)}$$

$$= \frac{2x-2-x}{(\ln 2) \times (x-1)}$$

$$= \frac{x-2}{(\ln 2) \times (x-1)}$$

$$9. y = x^{3/x}$$

$$\ln y = \frac{3}{x} \ln x \quad 2x^{-1}$$

$$\frac{1}{y} y' = \frac{3}{x} \cdot \frac{1}{x} - \frac{3 \ln x}{x^2}$$

$$\frac{1}{y} y' = \frac{3(1-\ln x)}{x^2}$$

$$y' = \frac{2x^{2/x}(1-\ln x)}{x^2} \text{ or } y' = 2(1-\ln x)x^{3/x-2}$$

$$10. y = (x-2)^{x+1}$$

$$\ln y = (x+1) \ln(x-2)$$

$$\frac{1}{y} y' = \frac{(x-1)}{x-2} + \ln(x-2)$$

$$y' = (x-2)^{x+1} \left(\frac{x-1}{x-2} + \ln(x-2) \right)$$

$$11. \int 3^{4x} dx \quad u = 4x \quad \frac{1}{4} du = dx$$

$$= \frac{1}{4} \int 3^u du = \frac{1}{4} \cdot \frac{3^u}{\ln 3} + C$$

$$= \frac{3^{4x}}{4 \ln 3} + C$$

$$12. \int (x^2 + 2^{-x}) dx$$

$$\frac{x^3}{3} - \frac{2^{-x}}{\ln 2} + C$$

$$13. \int x(5^{-x^2}) dx$$

$$u = -x^2$$

$$-1 du = dx$$

$$-\frac{1}{2} \int 5^u du$$

$$-\frac{1}{2} \frac{5^u}{\ln 5} + C$$

$$-\frac{5^{-x^2}}{2 \ln 5} + C$$

$$14. \int \frac{3^{2x}}{1+3^{2x}} dx$$

$$u = 1+3^{2x}$$

$$du = 2(3^{2x}) \ln 3 dx$$

$$\frac{1}{2 \ln 3} du = 3^{2x} dx$$

$$= \frac{1}{2 \ln 3} \int \frac{1}{u} du$$

$$= \frac{1}{2 \ln 3} \ln |u| + C$$

$$= \frac{1}{2 \ln 3} \ln(1+3^{2x}) + C$$

$$= \frac{\ln(1+3^{2x})}{2 \ln 3} + C$$

$$15. \int_{-1}^2 2^x dx$$

$$= \frac{2^x}{\ln 2} \Big|_{-1}^2 = \frac{2^2}{\ln 2} - \frac{2^{-1}}{\ln 2}$$

$$= \frac{4 - \frac{1}{2}}{\ln 2}$$

$$= \frac{7/2}{\ln 2}$$

$$= \frac{7}{2 \ln 2}$$

$$16. \int_{-2}^2 4^{x^2} dx$$

$$u = x^2$$

$$2du = dx$$

$$2 \int_{-1}^1 4^u du$$

$$2 \cdot \frac{4^u}{\ln 4} \Big|_{-1}^1 = \frac{2}{\ln 4} (4 - 4^{-1})$$

$$= \frac{2}{\ln 4} \left(\frac{15}{4}\right) = \frac{15}{2 \ln 4} \text{ or } \frac{15}{4 \ln 2}$$

$$17. \int_0^1 (5^x - 3^x) dx$$

$$\frac{5^x}{\ln 5} - \frac{3^x}{\ln 3} \Big|_0^1 = \left(\frac{5}{\ln 5} - \frac{3}{\ln 3}\right) - \left(\frac{1}{\ln 5} - \frac{1}{\ln 3}\right)$$

$$= \frac{4}{\ln 5} - \frac{2}{\ln 3}$$

$$18. \int_1^e (6^x - 2^x) dx$$

$$\frac{6^x}{\ln 6} - \frac{2^x}{\ln 2} \Big|_1^e$$

$$\frac{6^e}{\ln 6} - \frac{2^e}{\ln 2} - \left(\frac{6}{\ln 6} - \frac{2}{\ln 2}\right)$$

$$\frac{6^e - 6}{\ln 6} - \frac{2^e - 2}{\ln 2}$$