

6.1 to 6.5 Review #3

$$1. y = \ln(\ln 3x^2)$$

$$\frac{dy}{dx} = \frac{1}{\ln 3x^2} \cdot \frac{1}{3x^2} \cdot 6x$$

$$\frac{dy}{dx} = \frac{2}{x \ln 3x^2}$$

$$2. y = \ln\left(\frac{3x^2 - 3x + 2}{5x - 1}\right)$$

$$y = \ln(3x^2 - 3x + 2) - \ln(5x - 1)$$

$$y' = \frac{1}{3x^2 - 3x + 2} (6x - 3) - \frac{1}{5x - 1} \cdot 5$$

$$y' = \frac{6x - 3}{3x^2 - 3x + 2} - \frac{5}{5x - 1}$$

OR

$$y' = \frac{(5x-1)}{3x^2-3x+2} \cdot (5x-1)(6x-3) - (3x^2-3x+2)5$$

$$y' = \frac{(5x-1)(6x-3)}{(3x^2-3x+2)(5x-1)} - \frac{5(3x^2-3x+2)}{(3x^2-3x+2)(5x-1)}$$

$$y' = \frac{6x-3}{3x^2-3x+2} - \frac{5}{5x-1}$$

$$3. y = \ln \sqrt{x^2 - 4x - 7}$$

$$y = \frac{1}{2} \ln(x^2 - 4x - 7)$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2 - 4x - 7} \cdot 2x - 4$$

$$y' = \frac{2x - 4}{2(x^2 - 4x - 7)}$$

$$y' = \frac{x - 2}{x^2 - 4x - 7}$$

$$4. y = \ln\left(x \cdot \sqrt[3]{\frac{2x-1}{2x+1}}\right)$$

$$y = \ln x + \ln\left(\frac{2x-1}{2x+1}\right)^{1/3}$$

$$y = \ln x + \frac{1}{3}(\ln(2x-1) - \ln(2x+1))$$

$$y' = \frac{1}{x} + \frac{1}{3(2x-1)} \cdot 2 - \frac{1}{3(2x+1)} \cdot 2$$

$$y' = \frac{1}{x} + \frac{2}{3(2x-1)} - \frac{2}{3(2x+1)}$$

you would combine w/ common denominators.

$$5. y = -2x^2 + \ln x - 1 \quad \text{at } (1, -3)$$

$$y' = -4x + \frac{1}{x}$$

$$y'(1) = -4(1) + \frac{1}{1} = -4 + 1 = -3$$

$$y + 3 = 3(x-1)$$

$$y + 3 = 3x - 3$$

$$y = 3x - 6$$

$$6. y = (\sin x)^{3x^2}$$

$$\ln y = \ln(\sin x)^{3x^2}$$

$$\ln y = 3x^2 \ln(\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = 6x \cdot \ln(\sin x) + 3x^2 \cdot \frac{1}{\sin x} \cdot 6x \cot x$$

$$\frac{1}{y} \frac{dy}{dx} = 6x \ln(\sin x) + 3x^2 \cot x$$

$$\frac{dy}{dx} = (\sin x)^{3x^2} (6x \ln(\sin x) + 3x^2 \cot x)$$

$$\frac{dy}{dx} = 3x (\sin x)^{3x^2} (2x \ln(\sin x) + x \cot x)$$

$$7. y = \sqrt{\frac{x^2-2}{x^2+2}}$$

Method 1

$$\ln y = \ln \left(\frac{x^2-2}{x^2+2} \right)^{1/2}$$

$$\ln y = \frac{1}{2} \ln(x^2-2) - \frac{1}{2} \ln(x^2+2)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2(x^2-2)}(2x) - \frac{1}{2(x^2+2)}(2x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{x^2-2} - \frac{x}{x^2+2}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x(x^2+2) - x(x^2-2)}{(x^2-2)(x^2+2)}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x^3 + 2x - x^3 + 2x}{(x^2-2)(x^2+2)}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{4x}{(x^2-2)(x^2+2)}$$

$$\frac{dy}{dx} = \sqrt{\frac{x^2-2}{x^2+2}} \left(\frac{4x}{(x^2-2)(x^2+2)} \right)$$

Method 2

$$y = \left(\frac{x^2-2}{x^2+2} \right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{x^2-2}{x^2+2} \right)^{-1/2} \left(\frac{(x^2+2)(2x) - (2x)(x^2-2)}{(x^2+2)^2} \right)$$

$$= \left(\frac{x^2+2}{x^2-2} \right)^{1/2} \left(\frac{x^3 + 2x - x^3 + 2x}{(x^2+2)^2} \right)$$

$$= \sqrt{\frac{x^2+2}{x^2-2}} \cdot \left(\frac{4x}{(x^2+2)^2} \right)$$

$$8. \int \frac{1}{x^2-4} dx \quad u = x^2-4 \\ du = 2x dx \\ \frac{1}{2} du = x dx$$

$$\frac{1}{2} \int \frac{1}{u} du$$

$$+ \ln|u| + C$$

$$\boxed{\frac{1}{2} \ln|x^2-4| + C}$$

$$9. \int_1^c \frac{(1+\ln x)^3}{x} dx \quad u = 1+\ln x \\ du = \frac{1}{x} dx$$

$$\int_1^2 u^3 du$$

$$u(e) = 1+\ln e = 1+1=2 \\ u(1) = 1+\ln 1 = 1+0=1$$

$$\left. \frac{u^4}{4} \right|_1^2$$

$$\frac{2^4}{4} - \frac{1^4}{4} = 4 - \frac{1}{4} = \boxed{15/4}$$

$$10. y = \frac{x}{e^{4x}}$$

$$y' = \frac{e^{4x}(1) - x \cdot 4e^{4x}}{(e^{4x})^2}$$

$$y' = \frac{e^{4x} - 4xe^{4x}}{(e^{4x})^2}$$

$$y' = \frac{e^{4x}(1-4x)}{(e^{4x})^2}$$

$$\boxed{y' = \frac{1-4x}{e^{4x}}}$$

$$11. xe^x + 8x - 3y = 0$$

$$1 \cdot e^x + xe^x + 8 - 3 \frac{dy}{dx} = 0$$

$$-3 \frac{dy}{dx} = -e^x - xe^x - 8$$

$$\boxed{\frac{dy}{dx} = \frac{e^x + xe^x + 8}{3}}$$

$$12. y = \ln(x+e^x)$$

$$y' = \frac{1}{x+e^x} (1+e^x)$$

$$\boxed{y' = \frac{1+e^x}{x+e^x}}$$

$$13. \int 2xe^{x^2} dx$$

$u = x^2$
 $du = 2x dx$

$$\int e^u du$$

$$e^u + C$$

$$\boxed{e^{x^2} + C}$$

$$14. \int \cos x \cdot e^{\sin x} dx$$

$u = \sin x$
 $du = \cos x dx$

$$\int e^u du$$

$$e^u + C$$

$$\boxed{e^{\sin x} + C}$$

$$15. \int_0^1 \frac{e^x}{2+e^x} dx$$

$u = 2+e^x$
 $du = e^x dx$

$$u(1) = 2+c^1 = 2+e$$

$$u(0) = 2+e^0 = 2+1=3$$

$$\int_3^2 \frac{1}{u} du$$

$$[\ln|u|]_3^{2+e}$$

$$\frac{\ln|2+e|- \ln 3}{\ln\left(\frac{2+e}{3}\right)}$$

$$16. \underline{y = 3^{x^2-2x}}$$

$$\boxed{y' = 3^{x^2-2x} (\ln 3)(2x-2)}$$

$$17. \underline{y = x b^{-x}}$$

$$y'_1 = 1 \cdot b^{-x} + x \cdot b^{-x} (\ln b)(-1)$$

$$y'_1 = b^{-x} - x b^{-x} (\ln b)$$

$$\boxed{y' = b^{-x} (1-x \ln b)}$$

$$18. \underline{y = \sqrt{e^{3x}-4x}}$$

$$y'_1 = \frac{1}{2} (e^{3x}-4x)^{-1/2} (3e^{3x}-4)$$

$$\boxed{y' = \frac{3e^{3x}-4}{2\sqrt{e^{3x}-4x}}}$$

$$19. \underline{y = e^x (\sin x - \cos x)}$$

$$y'_1 = e^x (\sin x - \cos x) + e^x (-\cos x + \sin x)$$

$$y'_1 = e^x (\sin x - \cos x) + e^x (\sin x + \cos x)$$

$$y'_1 = e^x \sin x - e^x \cos x + e^x \sin x + e^x \cos x$$

$$\boxed{y' = 2e^x \sin x}$$

$$20. \underline{y = 10^{x^2-\sin x}}$$

$$\boxed{y' = 10^{x^2-\sin x} (\ln 10) (2x - \cos x)}$$

$$21. \underline{y = \pi^x}$$

$$y'_1 = \pi^x (\ln \pi) u$$

$$\boxed{y' = \pi^x (\ln \pi)}$$

$$22. \int \frac{e^{\tan x}}{\cos^2 x} dx$$

$u = \tan x$
 $du = \sec^2 x dx$
 $du = \frac{1}{\cos^2 x} dx$

$$\int e^u du$$

$$e^u + C$$

$$\boxed{e^{\tan x} + C}$$

$$23. \int_1^9 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

$u = \sqrt{x}$
 $du = \frac{1}{2\sqrt{x}} dx$

$$\int_1^3 e^u du$$

$$e^u \Big|_1^3$$

$$u(3)=3$$

$$u(1)=1$$

$$c^3 - c^1 = \boxed{c^3 - c}$$

$$24. \int_1^3 \frac{1}{x} dx$$

$$\ln|x| \Big|_1^3$$

$$\ln 3 - \ln 1$$

$$\boxed{\ln 3}$$

$$25. \int \frac{\log_4(5x)}{x} dx$$

$$\int \frac{\ln(5x)}{\ln 4 \cdot x} dx$$

$$\frac{1}{\ln 4} \int \frac{\ln 5x}{x} dx \quad u = \ln 5x \quad du = \frac{1}{5x} \cdot 5 dx$$

$$\frac{1}{\ln 4} \int u du \quad du = \frac{1}{x} dx$$

$$\frac{1}{\ln 4} \cdot \frac{u^2}{2} + C$$

$$\frac{1}{2 \ln 4} \ln^2(5x) + C$$

$$\boxed{\frac{\ln^2(5x)}{4 \ln 2} + C}$$

26.

$$7000 = 5000 e^{3k}$$

$$\frac{7}{5} = e^{3k}$$

$$\ln(\frac{7}{5}) = 3k$$

$$k \approx 0.112157$$

$$P = 5000 e^{0.112157 t}$$

$$\boxed{P \approx 92491.608}$$

$$1,000,600 = 5000 e^{0.112157 t}$$

$$200 = e^{0.112157 t}$$

$$\ln 200 = 0.112157 t$$

$$\boxed{t \approx 47.240 \text{ days}}$$

27.

$$\frac{1}{2} = 1 e^{5750k}$$

$$\frac{1}{2} = e^{5750k}$$

$$\ln \frac{1}{2} = 5750k$$

$$k \approx -0.000121$$

$$1 = 15 e^{-0.000121 t}$$

$$\frac{1}{15} = e^{-0.000121 t}$$

$$\ln \frac{1}{15} = -0.000121 t$$

$$\boxed{t \approx 22380.580 \text{ years}}$$