

## 2nd Semester Exam Review 2

$$1. \int 2xe^x dx$$

$$\begin{aligned} u &= 2x & v' &= e^x dx \\ u' &= 2dx & v &= e^x \end{aligned}$$

$$= 2xe^x - \int 2e^x dx$$

$$= 2xe^x - 2e^x + C$$

$$2. \int x \cos 2x dx$$

$$\begin{aligned} u &= x \\ u' &= dx \end{aligned}$$

$$\begin{aligned} v' &= \cos 2x dx \\ v &= \frac{\sin 2x}{2} \end{aligned}$$

$$= \frac{x \sin 2x}{2} - \frac{1}{2} \int \sin 2x dx$$

$$= \frac{x \sin 2x}{2} + \frac{1}{4} \cos 2x + C$$

$$3. \int x^2 \ln x dx$$

$$\begin{aligned} u &= \ln x \\ u' &= \frac{1}{x} dx \end{aligned}$$

$$\begin{aligned} v' &= x^2 dx \\ v &= \frac{x^3}{3} \end{aligned}$$

$$= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$$

$$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

$$4. \int_0^{\pi} x^2 \cos x dx$$

$$\begin{array}{r} x^2 \\ 2x \\ 2 \\ 0 \end{array} \quad \begin{array}{l} /+ \\ /- \\ /- \\ /+ \end{array} \quad \begin{array}{l} \cos x \\ \sin x \\ -\cos x \\ -\sin x \end{array}$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x \Big|_0^{\pi}$$

$$= \pi^2(0) + 2\pi(-1) - 2(0) = 0 - 0 + 0$$

$$= -2\pi$$

$$5. \int x^5 \sin x dx$$

$x^5$	$\downarrow$	$\sin x$
$5x^4$	$\downarrow$	$-\cos x$
$20x^3$	$\downarrow$	$-\sin x$
$60x^2$	$\downarrow$	$\cos x$
$120x$	$\downarrow$	$\sin x$
$120$	$\downarrow$	$-\cos x$
$0$	$\downarrow$	$-\sin x$

$$= -x^5 \cos x + 5x^4 \sin x + 20x^3 \cos x - 60x^2 \sin x - 120x \cos x + 120 \sin x$$

$$6. \int \frac{7x+39}{x^2+9x+14} dx = \int \frac{7x+39}{(x+7)(x+2)} dx$$

$$\frac{A}{x+7} + \frac{B}{x+2} = \frac{7x+39}{(x+7)(x+2)}$$

$$\begin{aligned} Ax+7A+Bx+7B &= 7x+39 \\ (A+B)x + 2A+7B &= 7x+39 \end{aligned}$$

$$\begin{aligned} A+B &= 7 & B &= 5 & A &= 2 \\ 2A+7B &= 39 \\ 2A+14 &= 14 \\ 5B &= 25 \end{aligned}$$

$$\int \left( \frac{2}{x+7} + \frac{5}{x+2} \right) dx$$

$$= 2 \ln|x+7| + 5 \ln|x+2| + C$$

$$7. \int \frac{x-1}{x^2+3x+2} dx = \int \frac{x-1}{(x+2)(x+1)} dx$$

$$\frac{A}{x+2} + \frac{B}{x+1} = \frac{x-1}{(x+2)(x+1)}$$

$$Ax+1 + Bx+2B = x-1$$

$$(A+B)x + A+2B = x-1$$

$$A+B=1$$

$$(-) A+2B=-1$$

$$-B=2$$

$$B=-2 \quad A=3$$

$$\int \left( \frac{3}{x+2} - \frac{2}{x+1} \right) dx$$

$$= 3 \ln|x+2| - 2 \ln|x+1| + C$$

$$8. \int \frac{2-y}{x^2+x} dx$$

$$\frac{A}{x} + \frac{B}{x+1} = \frac{2-y}{x(x+1)}$$

$$Ax+1A + Bx = 2-y$$

$$(A+B)x + A = 2-y$$

$$A+B=-1$$

$$A=2$$

$$B=-3$$

$$\int \left( \frac{2}{x} - \frac{3}{x+1} \right) dx$$

$$2 \ln|x| - 3 \ln|x+1| + C$$

$$9. \int \frac{dy}{(x+2)(x+3)}$$

$$\frac{A}{x+2} + \frac{B}{x+3} = \frac{dy}{(x+2)(x+3)}$$

$$Ax+3A + Bx+2B = 1$$

$$(A+B)x + 3A+2B = 1$$

$$2A+2B=0$$

$$3A+2B=1$$

$$10. \int \frac{2x^2+5x-1}{x^3+x^2-2x} dx$$

$$\int \frac{2x^2+5x-1}{x(x+2)(x-1)} dx$$

$$\frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-1} = \frac{2x^2+5x-1}{x(x+2)(x-1)}$$

$$A(x+2)(x-1) + Bx(x-1) + Cx(x+2) = \\ 4x^2 + Ax - 2A + Bx^2 - Bx + Cx^2 + 2Cx = \\ (A+B+C)x^2 + (A-B+2C)x - 2A = 2x^2 + 5x - 1$$

$$A+B+C=2 \quad -2A=-1 \quad A=\frac{1}{2}$$

$$A-B+2C=5$$

$$B+C=\frac{3}{2}$$

$$-B+2C=\frac{9}{2}$$

$$\frac{3C=6}{C=2}$$

$$B+2=\frac{3}{2}$$

$$B=-\frac{1}{2}$$

$$\int \left( \frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2} + \frac{2}{x-1} \right) dx$$

$$\frac{1}{2} \ln|x| - \frac{1}{2} \ln|x+2| + 2 \ln|x-1| + C$$

$$11. x(t)=t^2+1$$

$$y(t)=2t-1$$

$$\frac{dx}{dt}=2t \quad \frac{dy}{dt}=2$$

$$\frac{dy}{dx} = \frac{2}{2t} = \frac{1}{t} = t^{-1}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{t^2} \cdot \frac{1}{2t} = -\frac{1}{2t^3}$$

$$\int \left( \frac{1}{x+2} - \frac{1}{x+3} \right) dx$$

$$\ln|x+2| - \ln|x+3| + C$$

$$12. \quad x(t) = 2t^2 - 1$$

$$y(t) = 3t^3 + t$$

$$\frac{dx}{dt} = 4t \quad \frac{dy}{dt} = 9t^2 + 1$$

$$\frac{dy}{dx} = \frac{9t^2 + 1}{4t}$$

$$\frac{d^2y}{dx^2} = \frac{4t(18t) - 4(9t^2 + 1)}{16t^2} \cdot \frac{1}{4t}$$

$$= \frac{72t^2 - 36t^2 - 4}{64t^3}$$

$$= \frac{36t^2 - 4}{64t^3}$$

$$= \frac{9t^2 - 1}{16t^3}$$

$$13. \quad x(t) = 2\sec\theta - 3$$

$$y(t) = 4\tan\theta + 2$$

$$\frac{dx}{dt} = 2\sec\theta\tan\theta \quad \frac{dy}{dt} = 4\sec^2\theta$$

$$\frac{dy}{dx} = \frac{4\sec^2\theta}{2\sec\theta\tan\theta} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2 \frac{1}{\cos\theta} \cdot \frac{\csc\theta}{\sin\theta} = 2\csc^2\theta$$

$$\frac{d^2y}{dx^2} = \frac{-2\csc^2\theta}{2\sec\theta\tan\theta}$$

$$= \frac{-1}{\sin\theta} \cdot \frac{\csc\theta}{\sin\theta} \cdot \csc\theta \cdot \frac{\csc\theta}{\sin\theta}$$

$$= -\frac{\csc^3\theta}{\sin^3\theta} = -\cot^3\theta$$