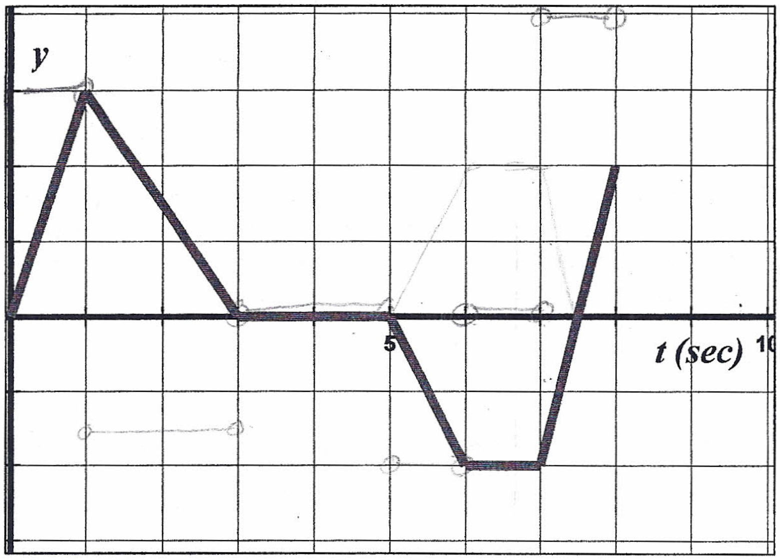
2.3 Rates of Change Group Review

AP Calculus

1. The accompanying figure shows the velocity $v = f(t)$ of a particle moving on a coordinate line.



a. When does the particle move forward? Move backward? *Speed* up? *Slow* down?

F: $0<t<3$ $7.5<t<8$
B: $5<t<7.5$
Sp: $0<t<1$ $5<t<6$ $7.5<t<8$
down: $1<t<3$ $7<t<7.5$

b. When is the particle's *acceleration* positive? Negative? Zero?

P: $0<t<1$ $7<t<8$ N: $1<t<3$ $5<t<6$

Z: $3<t<5$
$6<t<7$

c. When does the particle move at its greatest *speed*?

@ $t=1$

d. When does the particle stand still for more than an instant?

$3 \le t \le 5$

2. A rock thrown vertically upward from the surface of the moon at a velocity of 42 m/sec reaches a height of $s = 42t - 0.8t^2$ meters in t seconds.

a. Find the rock's velocity and acceleration at time t. $v(t) = s'(t) = 42 - 1.6t$ $a(t) = v'(t) = -1.6$

b. How long does it take the rock to reach its highest point? $v = 42 - 1.6t = 0$ max where s' is zero
$t = 26.25$ sec

c. How high does the rock go? $s(26.25) = 551.25$ m

d. How long does it take the rock to reach half its maximum height? $275.625 = 42t - 0.8t^2$ Q. Formula
$t = 44.812$ sec down $t \approx 7.658$ sec up

e. How long is the rock aloft? $42t - 0.8t^2 = 0$
$t \approx 52.5$ sec

3. Find the general formula for $F''(x)$ if $F(x) = xf(x)$ and f and f' are differentiable at x.

$F'(x) = xf'(x) + 1\,f(x)$
$= xf'(x) + f(x)$
$F''(x) = xf''(x) + 1 \cdot f'(x) + f'(x)$
$= xf''(x) + 2f'(x)$

4. Find a function $y = ax^2 + bx + c$ whose graph has an x-intercept of 1, a y-intercept of -2, and a tangent line with a slope of -1 at the y-intercept.

$(1,0)$ $(0,-2)$ $m=-1$ @ $(0,-2)$

$(1,0)$ $0=a+b+c$ $2=a+b$

$(0,-2)$ $-2=c$

$0=a+b-2$ $2=a-1$ $a=3$

$\boxed{y=3x^2-x-2}$

$y'=2ax+b$ ← Eq of m_{tan}

$2ax+b=-1$ @ $(0,-2)$

$2a(0)+b=-1$

$b=-1$

5. Find all points where f fails to be differentiable. Justify your answer.

A. $f(x) = |3x-2|$

$3x-2=0$

$x=2/3$

B. $g(x) = |x^2-4|$

$x^2-4=0$

$\boxed{x=\pm 2}$

6. According to ***Newton's Law of Cooling***, the rate of change of an object's temperature is proportional to the difference between the temperature of the object and that of the surrounding medium. The accompanying figure shows the graph of the temperature T (in degrees Fahrenheit) versus time t (in minutes) for a cup of coffee, initially with a temperature of $200°F$, that is allowed to cool in a room with a constant temperature of $75°F$.

A. Estimate T and dT/dt when $t = 10$ min.

$T \approx 120°$ $\frac{dT}{dt} \approx \frac{90-120}{20-10} = -30/10 \approx -3$

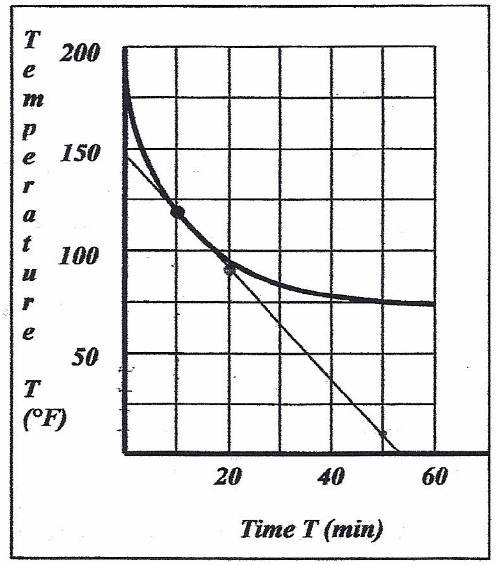
B. Newton's Law of Cooling can be expressed as $\frac{dT}{dt} = k(T - T_0)$ where k is the constant of proportionality and T_0 is the temperature (assumed constant) of the surrounding medium. Use the results in part (a) to estimate the value of k.

$-3 = k(120-75)$

$\frac{-3}{45} = \frac{45k}{45}$

$-\frac{1}{15} = k$

$k \approx -0.067$



7. Suppose that a product currently sells for \$25, with the price increasing at the rate of \$2 per year. At this current price, consumers will buy 150 thousand items, but the number sold is decreasing at the rate of 8 thousand per year. Revenue = quantity x price, since these quantities are changing in time, we write $R(t) = Q(t)P(t)$, where $R(t)$ is revenue, $Q(t)$ is quantity sold and $P(t)$ is the price, all at time t. At what rate is the total revenue increasing or decreasing?

$P(t)=25$ $Q(t)=150$

$P'(t)=2$ $Q'(t)=-8$

$R(t)=Q(t)P(t)$

$R'(t)=Q(t)P'(t)+Q'(t)P(t)$

$=150(2)+(-8)(25)$

$=300-200$

$R'(t)=100{,}000$ per year, revenue is increasing