

First Semester Review AP-Calc

Fri 6 ()²
Fri 13 3/5x

1. $y = \frac{2x+5}{3x-2}$

$$y' = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$$

$$y' = \frac{6x-4 - 6x-15}{(3x-2)^2}$$

$$y' = \frac{-19}{(3x-2)^2}$$

2. A.
 0 ≤ t ≤ 1 5 ≤ t ≤ 7 forward
 1 ≤ t ≤ 5 backward
 1 ≤ t ≤ 2 5 ≤ t ≤ 6 speed up
 0 ≤ t ≤ 1 3 ≤ t ≤ 5 slows down
 6 ≤ t ≤ 7

- B. Pos: $3 < t < 6$
 Neg: $0 < t < 2$ $6 < t < 7$
 Zeros: $2 < t < 3$ $7 < t < 9$

- C. $t=0$ and $2 \leq t \leq 3$

- D. $7 \leq t \leq 9$

3. A. $f(x) = x^2 + 2x - 1$ [0, 1]
 continuous on [0, 1]
 differentiable on (0, 1)

$$\begin{aligned} f(1) &= 2 \\ f(0) &= -1 \\ f(c) &= \frac{2+1}{1-0} = 3 \end{aligned}$$

$$2c+2 = 3$$

$$2c = 1$$

$$c = \frac{1}{2}$$

- B. No. Vertical tangent
 at $x=0$ $\frac{d}{dx} f(x)$ in $[1, 1]$

4. A. $t=2$, $t=6$, $t=10$
 B. $t=4$, $t=8$, $t=11$
 C. $0 < t < 2$ $6 < t < 10$
 D. $2 < t < 6$ $10 < t < 16$
 E. $4 < t < 8$ $11 < t < 16$
 F. $0 < t < 4$ $8 < t < 11$

5. $y = (\sin x)^{3x^3}$

$$\frac{dy}{dx} = 3x^2(\sin x)^{3x^2-1} (9x^2) = 27x^3(\sin x)^{3x^2-1}$$

$$6. \int \frac{(6x^2)}{(t-2x^3)^4} dx$$

$$u = -3-2x^3$$

$$du = -6x^2 dx$$

$$-du = 6x^2 dx$$

$$-\int u^{-2} du$$

$$+ \frac{1}{-3-2x^3} + C$$

$$7. f(x) = (x-5)^2$$

$$x = (y-5)^2$$

$$-5x = y-5$$

$$y = 5 + 5x = f^{-1}(x)$$

$$f(f^{-1}(x)) = (5 + 5x - 5)^2 = 5x^2 = x$$

$$f^{-1}(f(x)) = 5 + \sqrt{(x-5)^2} = 5+x-5 = x$$

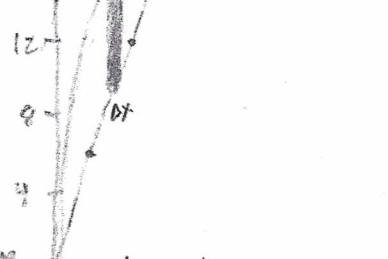
$$8. V = 6(t-1)(t-3) \quad t=1 \quad t=3$$

$$V = 6(t^2 - 4t + 3) \quad \text{pos } 0 \leq t \leq 1$$

$$V(t) = 6t^2 - 24t + 18 \quad \text{new } 1 \leq t \leq 3$$

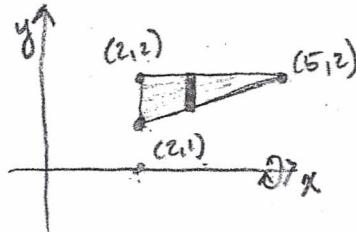
$$\begin{aligned} \text{Total dist} &= 6 \int_0^1 (t^2 - 4t + 3) dt - 6 \int_3^3 (t^2 - 4t + 3) dt \\ &= 6 \left(\frac{t^3}{3} - 2t^2 + 3t \right) \Big|_0^1 - 6 \left(\frac{t^3}{3} - 2t^2 + 3t \right) \Big|_1^3 \\ &\approx 6 \left(\frac{1}{3} - 2 + 3 \right) - 6 \left[(9 - 18 + 9) - \left(\frac{1}{3} - 2 + 3 \right) \right] \\ &= 8 + 8 = \boxed{16} \text{ meters. } \boxed{B} \end{aligned}$$

$$\begin{aligned} 9. 6x &= 12x - 2x^2 \\ 0 &= 6x - 2x^2 \\ 0 &= 2x(3-x) \\ x=0 & \quad x=3 \end{aligned}$$



$$\begin{aligned} V &= 2\pi \int_0^3 x(12x - 2x^2) - 6x dx \\ &= 2\pi \int_0^3 (6x^2 - 2x^3) dx \\ &= 2\pi \left(2x^3 - \frac{x^4}{2} \right) \Big|_0^3 = 2\pi (54 - 81/2) = \boxed{57\pi} \end{aligned}$$

10. $y=2$
 $m=\frac{1}{3}$
 $y-1=\frac{1}{3}(x-2)$
 $y-1=\frac{1}{3}x-\frac{2}{3}$



12. Disk Rx

$$V = \pi \int_0^1 (1-x^2)^2 dx$$

$$= 2\pi \int_0^1 (1-2x^2+x^4) dx$$

$$= 2\pi \left(x - \frac{2x^3}{3} + \frac{x^5}{5} \right) \Big|_0^1$$

$$= 2\pi \left(1 - 2/3 + 1/5 \right) = \boxed{\frac{16\pi}{15}}$$

13. $y=3\sqrt{x}$ $y=3$ $x=9$ about $y=3$

Washer dx

$$V = \pi \int_1^9 (3\sqrt{x}-3)^2 dx$$

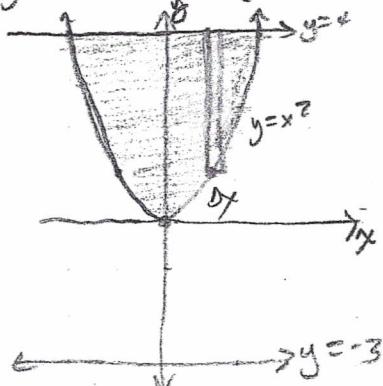
$$= \pi \int_1^9 (9x - 18x^{1/2} + 9) dx$$

$$= \pi \left(\frac{9x^2}{2} - 12x^{3/2} + 9x \right) \Big|_1^9$$

$$= \pi \left[\left(\frac{729}{2} - 324 + 81 \right) - \left(\frac{9}{2} - 12 + 9 \right) \right]$$

$$= \pi \left(243\frac{1}{2} - 3\frac{1}{2} \right) = \boxed{170\pi} \quad \boxed{B}$$

11. $y=x^2$ $y=4$ about $y=-3$



Washer dx

$$R(x)=7$$

$$r(x)=3+x^2$$

$$V = 2\pi \int_0^2 [7^2 - (3+x^2)^2] dx$$

$$= 2\pi \int_0^2 (49 - 9 - 6x^2 - x^4) dx$$

$$= 2\pi \int_0^2 (40 - 6x^2 - x^4) dx$$

$$= 2\pi \left(40x - 2x^3 - \frac{x^5}{5} \right) \Big|_0^2$$

$$= 2\pi (80 - 16 - 32/5) = \frac{576\pi}{5}$$

14.

$$A = \frac{d_1 \cdot d_2}{2}$$

$$A = \frac{(2 \cdot 3x^2)(2 \cdot 3x^2)}{2}$$

$$A = 18x^4$$

$$V = \int_0^3 18x^4 dx = \frac{18x^5}{5} \Big|_0^3 = \boxed{\frac{4374}{5}}$$

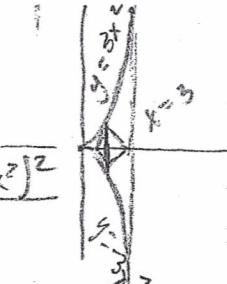
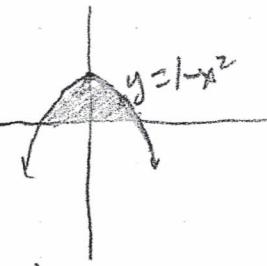
15. $y=1-x^2$ $y=-x-1$

$$A = \int_{-1}^2 (1-x^2 + x+1) dx$$

$$A = \int_{-1}^2 (2-x^2+x) dx$$

$$= 2x - \frac{x^3}{3} + \frac{x^2}{2} \Big|_{-1}^2$$

$$= 10/3 + 7/6 = \boxed{9/2}$$



$$16. \int_1^2 (x^2 + 1) dx$$

$$\frac{1}{2} \Delta x = \frac{1}{2}, \frac{2-1}{4} = \boxed{\frac{1}{4}} \quad \boxed{\frac{1}{2}} = \frac{1}{8}$$

$$A \approx \frac{1}{8} \left(f(1) + 2f\left(\frac{5}{4}\right) + 2f\left(\frac{3}{2}\right) + 2f\left(\frac{7}{4}\right) + f(2) \right)$$

$$\approx \frac{1}{8} \left(2 + 2\left(\frac{41}{16}\right) + 2\left(\frac{13}{4}\right) + 2\left(\frac{65}{16}\right) + 5 \right)$$

$$\approx \frac{1}{8} \left(2 + \frac{41}{8} + \frac{13}{2} + \frac{65}{8} + 5 \right)$$

$$\approx \frac{1}{8} \left(\frac{107}{4} \right) = \boxed{\frac{107}{32} \approx 3.344}$$

$$17. \int_1^{26/3} (-2x(5x^2+1)^{1/3}) dx \quad u=5x^2+1 \\ du=10x dx$$

$$-\frac{1}{5} \int_1^{27} u^{1/3} du$$

$$= -\frac{1}{5} \cdot \frac{3}{4} u^{4/3} \Big|_1^{27} = -\frac{3}{20} (81-1) =$$

$$= -\frac{3}{20} (80) = \boxed{-12}$$

$$18. \int_1^3 \frac{4x^3+2}{x^3} dx$$

$$= \int_1^3 (4 + 2x^{-3}) dx$$

$$= 4x - x^{-2} \Big|_1^3$$

$$= (12 - \frac{1}{9}) - (4 - 1)$$

$$= \frac{107}{9} - 3 = \frac{80}{9}$$

$$19. \int_0^{4\pi/2} \frac{-2\cos x}{\sqrt{9+\sin x}} dx \quad u=9+\sin x$$

$$du=\cos x dx$$

$$-2 \int_9^{10} u^{-1/2} du$$

$$= -2 \cdot 2u^{1/2} \Big|_9^{10}$$

$$\begin{matrix} 3.375 \\ 3.625 \\ 3.5 \end{matrix}$$

$$= -4(\sqrt{10} - 3) \\ = -4\sqrt{10} + 12 \boxed{}$$

$$20. F(x) = \int_{\sin x^2}^2 \frac{-1}{1-t^2} dt$$

$$= + \frac{1}{1-(\sin x^2)^2} (2x) \cos x^2$$

$$= \frac{2x \cos x^2}{1-\sin^2 x^2}$$

$$= \frac{2x \cos x^2}{\cos^2 x^2}$$

$$= \frac{2x}{\cos x^2} = \boxed{2x \sec x^2} \boxed{D}$$

$$21. y = x^3 + 2x + 1 \quad x=2 \quad y=3 \quad x-axis$$

$$A = \int_2^3 (x^3 + 2x + 1) dx$$

$$= \frac{x^4}{4} + x^2 + x \Big|_2^3$$

$$= \left(\frac{81}{4} + 9 + 3 \right) - (4 + 4 + 2)$$

$$= \frac{129}{4} - 10 = \boxed{\frac{89}{4}}$$

$$22. y = -x^2 + 7x + 30 \quad 3 \leq x \leq 7$$

$$Av(f) = \frac{1}{4} \int_3^7 (-x^2 + 7x + 30) dx \quad \frac{1}{b-a} = \frac{1}{4}$$

$$= \frac{1}{4} \left(-\frac{x^3}{3} + \frac{7x^2}{2} + 30x \right) \Big|_3^7$$

$$= \frac{1}{4} \left(\left(-\frac{343}{3} + \frac{343}{2} + 210 \right) - \left(-9 + \frac{63}{2} + 90 \right) \right)$$

$$= \frac{1}{4} \left(\frac{1603}{6} - \frac{225}{2} \right)$$

$$= \frac{1}{4} \left(\frac{464}{3} \right) = \boxed{\frac{116}{3}}$$

$$23. 1 \leq x \leq 2 \quad n=4 \quad \Delta x = \frac{2-1}{4} = \frac{1}{4} \quad y=x+2$$

$$M_{R,0,t} \quad A \approx \frac{1}{4} \left(f\left(\frac{9}{8}\right) + f\left(\frac{13}{8}\right) + f\left(\frac{17}{8}\right) + f\left(\frac{21}{8}\right) \right)$$

$$A \approx \frac{1}{4} (14) = \boxed{7/2}$$

$$\text{left} = \frac{27}{8} \quad 2 + 2 \frac{9}{8}$$

$$24. \int \sqrt{\tan 2x} \sec^2 2x dx \quad u = \tan 2x$$

$$= \frac{1}{2} \int u^{1/2} du$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = \boxed{\frac{1}{3} \sqrt{\tan^3 2x} + C}$$

$$25. \int \frac{3x}{\sqrt{4+3x^2}} dx, u = 4+3x^2 \\ du = 6x dx \\ \frac{1}{6} du = x dx$$

$$= \frac{1}{2} \int u^{-1/2} du$$

$$= \frac{1}{2} \cdot 2 u^{1/2} + C$$

$$= \boxed{\sqrt{4+3x^2} + C}$$

$$26. \int 8 \sin^3 \frac{4}{5}x \cos \frac{4}{5}x dx \quad u = \sin \frac{4}{5}x \\ \frac{4}{5} du = \cos \frac{4}{5}x dx$$

$$= 10 \int u^5 du = 10 \frac{u^6}{6} + C$$

$$= \boxed{\frac{5}{3} \sin^6 \frac{4}{5}x + C}$$

$$27. \int 4x^4 (-4-x^5)^3 dx \quad u = -4-x^5 \\ du = -5x^4 dx \\ -\frac{1}{5} du = x^4 dx$$

$$-\frac{4}{5} \int u^3 du$$

$$-\frac{4}{5} \cdot \frac{u^4}{4} + C$$

$$= \boxed{-\frac{1}{5} (-4-x^5)^4 + C}$$

$$28. (-2, -5) \quad m=2 \quad \frac{dy}{dx} = \frac{3x}{7}$$

$$\frac{dy}{dx} = \frac{3x^2}{14} + C \quad m = \frac{3x^2}{14} + C$$

$$\frac{dy}{dx} = \frac{3x^2}{14} + \frac{8}{7} \quad 2 = \frac{3(-2)^2}{14} + C$$

$$y = \frac{x^3}{14} + \frac{8}{7}x + C \quad \frac{8}{7} = 6$$

$$-5 = \frac{(-2)^3}{14} + \frac{8}{7}(-2) + C$$

$$-5 = -\frac{8}{14} - \frac{16}{7} + C \quad C = -15/7$$

$$y = \frac{x^3}{14} + \frac{8}{7}x - \frac{15}{7} \quad \boxed{B}$$

$$29. 3x^2 - 5y^2 = 9 \quad \frac{dy}{dt} = 12 \text{ m/sec}$$

Find $\frac{dy}{dt}$ when $x=3 \text{ m}$

$$6x \frac{dx}{dt} - 10y \frac{dy}{dt} = 0$$

$$6(3) \frac{dx}{dt} - 10(3\sqrt{5})(12) = 0 \quad -5y^2 = -18 \quad y^2 = 18/5$$

$$18 \frac{dx}{dt} - 360\sqrt{2}/5 = 0$$

$$18 \frac{dx}{dt} = 360\sqrt{2}/5$$

$$\frac{dx}{dt} = \frac{20\sqrt{2}}{5}$$

$$\boxed{\frac{dx}{dt} = 4\sqrt{10} \text{ m/sec}}$$

A

$$30. 3x^3 + 7 \cos(xy) = 3$$

$$9x^2 - 7 \sin(xy)(xy' + y) = 0$$

$$-7 \sin(xy)(xy' + y) = -9x^2$$

$$xy' + y = \frac{9x^2}{7 \sin(xy)}$$

$$xy' = \frac{9x^2}{7 \sin(xy)} - y$$

$$\boxed{\frac{dy}{dx} = \frac{9x}{7 \sin(xy)} - \frac{y}{x}}$$

$$31. 2xy + 9y^4 = 5x^3 + 2y$$

$$2xy' + 2y + 36y^3y' = 15x^2 + 2y'$$

$$2xy' + 36y^3y' - 2y' = 15x^2 - 2y$$

$$y'(2x + 36y^3 - 2) = 15x^2 - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{15x^2 - 2y}{2x + 36y^3 - 2}}$$

$$32. y = 24x^3 - 12x^2 - 70x + 3$$

$$y' = 72x^2 - 24x - 70$$

$$y' = 2(36x^2 - 12x - 35)$$

$$\therefore x = \frac{+42}{36} - \frac{30}{36}$$

$$\boxed{x = \frac{7}{6} - \frac{5}{6}}$$

$$33. f(x) = -3x - 5x^2 \Rightarrow x = 2$$

$$f'(x) = -3 + 5x^{-2}$$

$$y = -3(2) - \frac{5}{2}$$

$$f'(2) = -3 + 5\left(\frac{1}{4}\right)$$

$$y = -\frac{17}{2}$$

$$f'(2) = -\frac{7}{4} = m$$

$$y + \frac{17}{2} = -\frac{7}{4}(x - 2)$$

$$y + \frac{17}{2} = -\frac{7}{4}x + \frac{7}{2}$$

$$y = -\frac{7}{4}x - 5$$

$$\frac{7}{4}x + y = 5$$

$$\boxed{7x + 4y = -20 \quad B}$$

34.

$$f(x) = \begin{cases} 18 & x < -3 \\ 2x^2 & -3 \leq x \leq 2 \\ -2x+13 & x > 2 \end{cases}$$

False Discontinuous at $x=2$

$$35. f(-4) \quad f(x) = \frac{x^2 - 36}{x^2 + 16x + 24} \Rightarrow x = -4$$

$$f(x) = \frac{(x+6)(x-6)}{(x+6)(x+4)}$$

$$f(x) = \frac{x-6}{x+4}$$

$$\lim_{x \rightarrow -6} \frac{x-6}{x+4} = \frac{-6-6}{-6+4} = \frac{-12}{-2} = 6$$

$$f(x) = \begin{cases} \frac{x^2 - 36}{x^2 + 16x + 24}, & x \neq -6 \\ 6, & x = -6 \end{cases}$$

$$36. \lim_{x \rightarrow \infty} \frac{3x^3 + 2x + 3}{-4x^3 - 5x + 5} = \boxed{-\frac{3}{4}} \quad C$$

$$37. \int \frac{3x^4}{(-4-3x^5)^3} dx$$

$$u = -4 - 3x^5 \\ du = -15x^4 dx \\ -\frac{1}{15} du = x^4 dx$$

$$= -\frac{1}{5} \int u^{-3} du$$

$$= \frac{1}{10} u^{-2} + C$$

$$= \frac{1}{10} (-4-3x^5)^{-2} + C$$

$$= \frac{1}{10(-4-3x^5)^2} + C$$