AP Calculus Mid-Term Review 2

1. $f(x)=\sin\left(\frac{x}{2}\right) \quad \frac{\pi}{2}<x<\frac{3\pi}{2}$

$$\frac{f\left(\frac{3\pi}{2}\right)-f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2}-\frac{\pi}{2}}=\frac{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}}{\pi}=\frac{0}{\pi}=0$$

$f'(x)=\frac{1}{2}\cos x$

$\frac{1}{2}\cos\frac{x}{2}=0 \quad \cos\frac{x}{2}=0 \quad \frac{x}{2}=\cos^{-1}0$

$\frac{x}{2}=\frac{\pi}{2}, \frac{3\pi}{2}$

$x=\pi \quad x=3\pi$

2. $f'(x)=\cos x \quad f\left(\frac{\pi}{2}\right)=3$

$f(x)=\int \cos x\,dx$

$f(x)=\sin x+C$

$3=\sin\frac{\pi}{2}+C$

$3=1+C$

$C=2$

$f(x)=\sin x+2$

3. $f(x)=(x^2-2x-1)^{2/3}$

$f'(x)=\frac{2}{3}(x^2-2x-1)^{-1/3}(2x-2)$

$f'(x)=\frac{4x-4}{3(x^2-2x-1)^{1/3}}$

$f'(0)=\frac{-4}{3(-1)^{1/3}}=\frac{-4}{-3}=\boxed{4/3}$

4. $\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=\boxed{-\sin x}$

5. $f'(1)=18 \quad f(x)=3x^2-12x+9$

$f'(x)=6x-12$

$6C-12=18$

$\boxed{C=5}$

6. $y=\frac{ax+b}{x+c}$

$y=2$

$x=-3$

$y=\frac{2x+b}{x+3}$

$a+c=2+(-3)=5$

7. $\int \frac{3x^2}{\sqrt{x^3+1}}dx \quad u=x^3+1 \quad du=3x^2dx$

$\int u^{-1/2}du$

$2u^{1/2}+C$

$\boxed{2\sqrt{x^3+1}+C}$

8. $\lim_{n\to\infty}\frac{4n^3-5n}{n^3-3n^2-1}$

$\lim_{n\to\infty}\frac{\frac{4n^3}{n^3}-\frac{5n}{n^3}}{\frac{n^3}{n^3}-\frac{3n^2}{n^3}-\frac{1}{n^3}}=\frac{4}{1}=\boxed{4}$

9. $x^3+4xy+3y^2=14$

$3x^2+4x\frac{dy}{dx}+4y+6y\frac{dy}{dx}=0$

$(4x+6y)\frac{dy}{dx}=-3x^2-4y$

$\boxed{\frac{dy}{dx}=-\frac{3x^2+4y}{4x+6y}}$

10. $f(x)=\frac{x^2-9}{x-3}=x+3 \quad$ or $\quad f'(x)=1$

$f'(x)=\frac{(x-3)(2x)-(x^2-9)(1)}{(x-3)^2}$

$f'(x)=\frac{2x^2-6x-x^2+9}{(x-3)^2}$

$f'(x)=\frac{x^2-6x+9}{(x-3)^2}$

$f'(-3)=\frac{9+18+9}{36}=1$

11. $y=\frac{3x-3}{2x-1} \quad y'(2)=\frac{(3)(3)-(3)(2)}{3^2}$

$y'=\frac{(2x-1)(3)-(3x-3)(2)}{(2x-1)^2} \quad y'(2)=\frac{9-6}{9}=3/9$

$\boxed{y-1=\frac{1}{3}(x-2)} \quad \leftarrow y'(2)=1/3$

12. $y = \tan x + \sec x$

$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$

13. $\int (x-2)^2 dx$

$= \int (x^2 - 4x + 4) dx$

$= \frac{x^3}{3} - 2x^2 + 4x + C$

14. $\int_a^b c x dx = a^2 - b^2$

$c \int_a^b x dx = a^2 - b^2$

$c\left(\frac{x^2}{2}\right)\Big]_a^b = a^2 - b^2$

$\frac{c}{2}\left(b^2 - a^2\right) = a^2 - b^2$

$\frac{c}{2} = \frac{a^2 - b^2}{b^2 - a^2}$

$\frac{c}{2} = -1$

$\boxed{c = -2}$

15. $\frac{d}{dx}\int_0^{x^2} \sin u \, du = \sin x^2 (2x)$

$\boxed{= 2x \sin x^2}$

16. ① ⓐ $\frac{dy}{dt} = -3$ ft/min

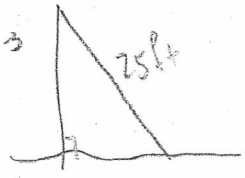
ⓑ $x^2 + y^2 = 25^2$

$x^2 + y^2 = 0$

ⓒ $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

ⓓ $24\frac{dx}{dt} + 7(-3) = 0$

$24\frac{dx}{dt} = 21$



$\frac{dy}{dt} = -3$

Find $\frac{dx}{dt}$ when $y = 7$

$x^2 + 7^2 = 25^2$

$x^2 + 49 = 625$

$x = 24$

$\frac{dx}{dt} = \frac{7}{8}$ ft/min

17. $f(x) = kx^2$

$f'(x) = 0$

$f'(2) = 0$

18. A. yes $f(-1) = -1$

B. yes $\lim_{x \to -1^+} = -1$

C. yes

D. yes

E. yes endpoint

19. $y = x\sqrt{4 - x^2}$

$A(x) = \int_0^2 x\sqrt{4-x^2}\,dx$ $u = 4 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$a(2) = 0$

$u(0) = 4$

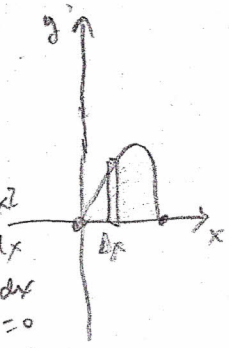
$= -\frac{1}{2}\int_4^0 u^{1/2}\,du$

$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2}\Big]_4^0$

$= -\frac{1}{3}u^{3/2}\Big]_4^0$

$= -\frac{1}{3}(0^{3/2} - 4^{3/2})$

$= -\frac{1}{3}(-8) = 8/3$



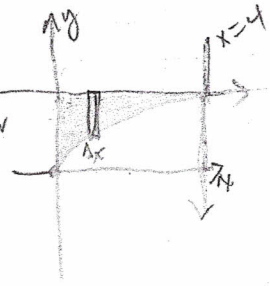
20. $y = \sqrt{x}$ $y = 2$ $x = 0$

20A. $V = \pi\int_0^4 (2^2 - \sqrt{x}^2)\,dx$ Washer

$V = \pi\int_0^4 (4 - x)\,dx$

$= \pi\left(4x - \frac{x^2}{2}\right)\Big]_0^4$

$= \pi(16 - 8) = 8\pi$



B. $V = 2\pi\int_0^4 x(2 - \sqrt{x})\,dx$ shell

$= 2\pi\int_0^4 (2x - x^{3/2})\,dx$

$= 2\pi\left(x^2 - \frac{2}{5}x^{5/2}\right)\Big]_0^4$

$= 2\pi\left(16 - \frac{2}{5} \cdot 32\right) = \boxed{\frac{32\pi}{5}}$

20.C $V = 2\pi \int_0^4 (4-x)(2-\sqrt{x})\,dx$ shell

$V = 2\pi \int_0^4 (8 - 4x^{1/2} - 2x + x^{3/2})\,dx$

$V = 2\pi \left(8x - 4\cdot\frac{2}{3}x^{3/2} - x^2 + \frac{2}{5}x^{5/2}\right)\Big|_0^4$

$V = 2\pi(32 - 64/3 - 16 + 64/5)$

$\boxed{V = \frac{224\pi}{15}}$

20.D. $V = \pi \int_0^4 (2-\sqrt{x})^2\,dx$ Disk

$= \pi \int_0^4 (4 - 4x^{1/2} + x)\,dx$

$= \pi \left(4x - 4\cdot\frac{2}{3}x^{3/2} + \frac{x^2}{2}\right)\Big]_0^4$

$= \pi\left(16 - 4\cdot\frac{2}{3}\cdot 8 + 8\right)$

$= \pi(16 - 64/3 + 8)$

$= \boxed{\frac{8\pi}{3}}$

21.A. $t = 8$

B. $t = 10$ Speed $|v(t)|$

C. $s(2) = \int_0^2 v(t)\,dt = \frac{1}{2}(2)(3) = 3$

$s'(2) = v(2) = 3$

D. $t = 2$, $t = 5$ and $t = 10$

E. displacement $=$

$$\int_0^{11} v(t)\,dt = \tfrac{1}{2}(2)(3) + \tfrac{1}{2}(1)(3+2) + (1)(2) + \tfrac{1}{2}(1)(2+1) + \tfrac{1}{2}(1)(2+1) + 1(2) + \tfrac{1}{2}(1)(2) - \tfrac{1}{2}(2)(4) - \tfrac{1}{2}(1)(4+3)$$

$$= \left(3 + 5/2 + 2 + \tfrac{3}{2} + \tfrac{3}{2} + 2 + 1\right) + \left(-4 - \tfrac{7}{2}\right) = \left(\tfrac{27}{2}\right) + \left(-15/2\right) = 6$$

Total Distance $= \int_0^8 v(t)\,dt - \int_8^{11} v(t)\,dt = \frac{27}{2} - \left(-\frac{15}{2}\right) = \frac{42}{2} = 21$

1st Semester Review This page is for Review #1

#2 Based on the velocity function - still look at answer key.

A. Moving Forward
look for $v(t) > 0 \rightarrow$ pos.

Backward
$v(t) < 0 \rightarrow$ neg.

Speeding Up
$v(t)$ moving away from x-axis

Slowing down
$v(t)$ moving towards x-axis

B. Acceleration Pos:
$v(t)$ has a pos. slope

Accel. Neg.
$v(t)$ has a neg. slope

Accel. Zero:
$v(t)$ is constant (horizontal)

C. Greatest Speed
where $v(t)$ is farthest from x-axis

D. Standing Still for more than an instant
where $v(t) = 0$ over an interval

*Additional Info

Changes directions
where $v(t)$ changes sign